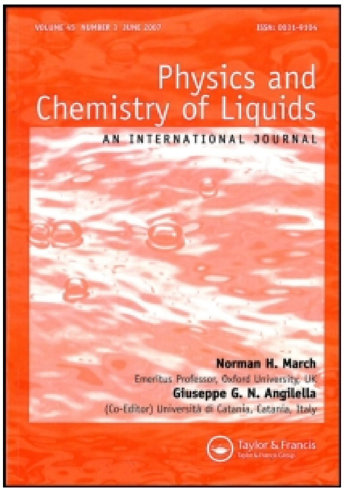

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N. V. K. Dutt[a]
[a] Chemical Engineering Division, Indian Institute of Chemical Engineering, Hyderabad, India

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DETERMINATION OF LOSS COEFFICIENTS FOR THE ENTRY REGION FLOW OF VISCO-ELASTIC FLUIDS

N. V. K. DUTT

Chemical Engineering Division, Indian Institute of Chemical Engineering, Hyderabad - 500 007, India

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Experimental values of the loss coefficients C_{VE} for the flow of visco-elastic fluids through an abrupt 2:1 pipe contraction have been correlated with recoverable shear, RS. Further, the elastic component of the loss coefficient has been estimated from the Hookean shear modulus, G, defined in terms of the shear stress and the normal stress difference. The relationship between C_{VE} and RS has been further tested by means of jet thrust data. In view of the limited data available, further experimentation is needed for testing the concepts proposed in this work.

Keywords: Visco-elasticity; loss coefficient; Hookean shear modulus; jet thrust data; recoverable shear

INTRODUCTION

Knowledge of the frictional losses in excess of the fully developed flow of Newtonian and non-Newtonian fluids through pipe contractions is essential for design purposes. The excess frictional losses represented by the loss coefficient, C, was defined by Skelland [1] by the equation, for the flow of fluids through pipe contractions:

$$\frac{P_0 - p_2}{\rho \bar{v}_2^2 / g_c} = 32 \frac{L_2/R_2}{R_{e,2}} + C \qquad (1)$$

The first and second terms of Eqn. (1) denote the dimensionless pressure drop and length and $P_{2,\rho}$, V_2, R_2 and $R_{e,2}$ denote the pressure, fluid density, average fluid velocity in the down stream pipe, axial position from the entrance in the smaller pipe, radius and the down stream pipe Reynolds number respectively.

$R_{e,2}$ is defined by the equation:

$$R_{e,2} = \frac{D_2^n V_2^{2-n}}{K 8^{n-1}\left((3n+1)/(4n)\right)^n} \qquad (2)$$

where n and K are the flow behaviour index and consistency index respectively.

In contrast to the purely viscous fluid flow wherein the energy is dissipated due to friction, in the case of visco-elastic fluids, the energy supplied is partly dissipated and the remainder manifests itself as recovery due to the elastic nature of the fluid. Hence, in the case of visco-elastic fluids flowing through pipe lines, the loss coefficients, C_{VE} will be less than that corresponding to the purely viscous value, C_{PV}. In the present paper, correlations on the loss coefficients determined for visco-elastic liquids in terms of the recoverable shear, RS and Hookean shear modulus, G have been proposed.

EXPERIMENTAL

The experimental set up used for the determination of the loss coefficient, C_{VE}, in the case of visco-elastic fluids is the same as that reported by Dutt [2] and Boger and Ramamurthy [3]. From the axial pressure drop ($P_0 - P_2$) measurements (P_0 and P_2 being the pressures at the contraction "o" and at the position located at a distance L_2 in the downstream pipe from the contraction), C_{VE} is calculated as the intercept obtained from the plot of the dimensionless pressure drop vs dimensionless axial length (terms 1 and 2 of Eqn. (1)). Details of the flow system can be obtained from Refs. (2) and (3).

RESULTS AND DISCUSSION

The liquids used in the flow system for the determination of the loss coefficients were characterized by an R-16 Weissenberg Rheogoniometer for the presence of normal stresses. Liquids with measurable normal stresses are categorized as visco-elastic liquids. In contrast, fluids which did not exhibit measurable normal stresses have been classified as purely viscous fluids.

The loss coefficients of the visco-elastic Methocel (aqueous solutions of a cellulose gum manufactured by Dow Chemical Co.) solutions, C_{VE}, as determined from the flow system consisting of a 2:1 pipe contractions are reported in Table I along with their flow behaviour indices, n.

For the same value of the flow behaviour index n, purely viscous liquids exhibit higher values of loss coefficient (C_{PV}) compared to visco-elastic liquids. The difference between C_{PV} and the corresponding value for visco-elastic liquids, C_{VE}, should represent the value due to the elastic nature of the liquid. Hence, an attempt has been made to investigate the relationship between ($C_{PV}-C_{VE}$) and the primary recoverable shear, RS_1, defined as the ratio of the first normal stress difference, ($P_{11}-P_{22}$) and the shear stress, T_{12}.

For the visco-elastic Methocel solutions under investigation in the present work, primary recoverable shear, RS_1 as a function of the shear rate, r has been determined on the R-16 Weissenberg Rheogoniometer using a Cone and Plate configuration. The average of the values of RS_1 determined for each of the liquids under study over the shear range of 73.5 Sec^{-1} to 1468 Sec^{-1} are reported in Table I.

In a recent publication, Dutt [2], has reported the equation:

$$C_{PV} = 1.8148(n(1-\beta))^{0.6604} \tag{3}$$

TABLE I Testing of equation for the methocel visco-elastic solutions in water $C_{PV}-C_{VE} = 0.19RS_1$ for Methocel Visco-elastic solutions in water

Polymer	*n*	C_{PV} *(Eqn. 3)*	$C_{PV}-C_{VE}$	RS_1	*% Dev. On* $C_{PV}-C_{VE}$	*Ref.*
M-4	0.510	0.7375	0.7375	2.66	31.5	2
M-5	0.587	0.8093	0.8093	3.51	17.6	2
M-6	0.515	0.7423	0.7423	4.57	−15.4	2
M-9	0.516	0.7432	0.7432	3.35	14.4	3
M-11	0.465	0.6939	0.6939	3.40	6.9	3
M-12	0.400	0.6282	0.6282	3.50	−5.9	3
M-7	0.541	0.7668	0.1468	1.30	−16.5	3
M-8	0.538	0.7640	0.1240	1.00	−53.2	3
M-9	0.470	0.6988	0.2988	0.90	17.3	3
Over all					19.9	

for the calculation of C_{PV}, with β denoting the ratio of the diameter of the downstream pipe to that of the upstream pipe. The values of C_{PV} calculated for a 2:1 pipe contraction with $\beta = 0.4985$ using Eqn. (3) are also reported in Table I. At zero shear rate ($r = 0$), both C_{PV} and C_{VE} will be equal to zero. Hence, a relationship of the type

$$C_{PV} - C_{VE} = A \ RS_1 \tag{4}$$

can be anticipated. The % deviation between calculated value of ($C_{PV} - C_{VE}$) using an optimum value of $A = 0.19$ for the liquids studied are reported in Table I. The average absolute deviation, e for the set of 9 liquids reported in the Table is 19.9%. The accuracy of the relationship could further be improved by the inclusion of the data on the secondary recoverable shear, RS_2 defined as the ratio of the secondary normal stress difference, ($P_{22} - P_{33}$) and the shear stress, T_{12}. The data on ($P_{22} - P_{33}$) can be calculated as the difference between the normal stresses measured by using a parallel platen system and those measured by the Cone and Platen system at the same shear rates. For the fluids exhibiting significant elastic forces, Dutt [2] and Boger and Ramamurthy [3] reported the flow in the downstream pipe to be fully developed, without exhibiting the entrance losses. In such cases, C_{VE} becomes zero and the frictional losses occurring in the upstream pipe in the form of loss coefficient assume a maximum corresponding to the value of C_{PV} estimated from the flow behaviour index n and the pipe contraction ratio, B using the equation (3).

In the case of the fluids with predominant elastic effects, Sylvester and Rosen [4] proposed the relationship between the elastic portion of the pipe line entrance pressure loss, ΔP_E, and the Hookean shear modulus, G, by means of the relationship

$$\Delta P_E = \frac{T_w^2}{G} f(n) \tag{5}$$

where T_w is the wall shear stress and $f(n)$ is defined in terms of flow behaviour index, n by

$$f(n) = \frac{1}{4}\left(\frac{3n+1}{5n+1}\right) \tag{6}$$

Hence, in the case of fluids with significant elastic effects, a relationship between C_{PV} and G can be anticipated.

Values of G for separan solutions (aqueous solutions of Poly acrylamide) were reported by Sylvester and Rosen [4]. In the case of CMC (Carboxyl Methyl Cellulose) solutions, G-values have been estimated from the plots of ΔP_E vs T_w^2 reported by the authors using the equations (5) and (6). For Methocel solutions (aqueous solutions of a cellulose gum) M-4 to M-10, G-values have been estimated from the data reported by Dutt [2] and Boger and Rama Murthy [3]. The values of G and C_{PV} (estimated from equation (3)) for several polymer solutions are reported in Table II. The data could be represented by the equation (7).

$$C = 0.86 - 0.00025\,G \tag{7}$$

with an average absolute deviation ($\bar{e}$) of 13.4% over a set of 12 points.

The concept of expressing the loss coefficient in terms of the recoverable shear could also be applied to the data on the flow of aqueous polymer solutions through a jet at a shear rate of 10^5 Sec^{-1} reported by Oliver [5]. Using the data on the wall normal stress difference ($P_{11}-P_{22}$) and shear stress (T_{12}), recoverable shears, RS were calculated and reported in Table III. The loss coefficient C in terms of the Hooekan shear modulus G has been estimated by the

TABLE II Testing of the equation (7) for several polymer solutions in water

Polymer	*n*	*G Dyn/Cm²*	*C_{PV} (Eqn. 3)*	*% Dev. on (7)*	*Ref.*
M-4	0.510	12.1	0.7375	−4.5	2
M-5	0.587	24.0	0.7423	−15.1	2
M-6	0.515	14.0	0.8093	−5.8	2
M-9	0.516	20.0	0.7432	−15.0	3
M-11	0.465	34.2	0.6939	−22.7	3
M-12	0.400	37.9	0.6282	−35.4	3
CMC-1	0.505	10.9	1.0583	19.0	4
CMC-2	0.395	14.1	0.8998	4.8	4
S-1	0.462	110.3	0.9979	16.6	4
S-2	0.365	206.8	0.8541	5.4	4
S-3	0.356	468.9	0.8401	11.6	4
S-4	0.289	655.0	0.7321	4.9	4
Over all				13.4	

M: Methocel; CMC: Carboxyl methyl cellulose; S: Poly acrylamide.

TABLE III Testing of jet thrust data*

Polymer	*n*	$C_{PV}-C_{VE}$	C_{PV}	*RS*	% *Dev.*
1% carbopol	0.71	0.8289	0.3588	6.22	6.9
1% SCMC	0.62	0.8244	0.3173	8.57	14.6
2% SCMC	0.54	0.7933	0.2795	9.29	16.0
0.5% ET	0.92	0.8330	0.4487	23.33	−5.0
0.01% ET	1.0	0.8560	0.3747	17.65	−10.7
0.01% polyox	1.0	0.8579	0.3766	33.53	−1.7
.05% polyox	1.0	0.8560	0.3747	39.47	−1.0
Over all					8.0

*Data at 10^5 sec^{-1} shear rate.

rearrangement of Equations (5) and (6) in terms of ΔP_E, taken to be equal to $(P_{11}-P_{22})_w$. This re-arrangement can be expressed into the form

$$G=\frac{1}{4}\left(\frac{3n+1}{5n+1}\right)\frac{T_w^2}{(P_{11}-P_{22})_w} \tag{8}$$

The visco-elastic portion of the loss coefficient, C_{VE} is calculated as the difference between C estimated from equation (7) and the purely viscous loss coefficient, C_{PV} from equation (3). The value of the contraction ratio β (for substitution into equation (3)) was calculated from the expression

$$\beta=\sqrt{\frac{2n+1}{3n+1}} \tag{9}$$

reported by Oliver [5].

In Table III, the data on the polymers with their flow behaviour indices (n) recoverable shear (RS) and the elastic portion of the loss coefficient, C_{VE} calculated as the difference of the loss coefficients calculated from equations (3) and (7) are tabulated. The data could be represented by the equation

$$C_{VE}=0.45-0.002\ \mathrm{RS} \tag{10}$$

with an average absolute deviation of 7%.

CONCLUSION

The results reported in the present investigation suggest reasonable correlations between the visco-elastic loss coefficients and the recoverable shears.

NOMENCLATURE

C	loss coefficient
D	diameter of pipe
G	Hookean Shear modulus defined by eqn. (8)
K	consistency index, given in eqn. (2)
L	axial position from the contraction, "o"
n	flow behaviour index
P	pressure
ΔP_E	elastic portion of the entrance loss defined by eqn. (5)
$P_{11}-P_{22}$	first (or primary) normal stress difference
$P_{22}-P_{33}$	secondary normal stress difference
R	radius of the pipe
Re	generalized Reynolds number defined by eqn. (2)
RS	recoverable shear, ratio of $(P_{11}-P_{22})$ to T_w
Rs1	primary recoverable shear, ratio of $(P_{11}-P_{22})$ to T_{12}
Rs2	secondary recoverable shear, ratio of $(P_{22}-P_{33})$ to T_{12}

Greek Symbols

β	ratio of the diameters of downstream and upstream pipes
r	shear rate
ρ	fluid density
T_{12}	shear stress

Suffixes

0	condition at the contraction
2	condition at axial position from contraction
PV	for the purely viscous fluid
VE	for the visco elastic fluid
w	condition at the wall

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